Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
A		2107028

Cellulose decomposition problem based on the relationship model of influence factors

As a key step in the carbon cycle, the decomposition of plant material and woody fibers is of great significance. This process is based on the decomposition of cellulose, hemicellulose, lignin, etc. by saprophytic fungi. To describe the interaction between various fungi and the effects of environmental and substrate quality on lignocellulosic decomposition, we built a process model with time as its only variable. It is called **the relationship model of influence factors**. Considering that the decomposition of lignin fibers by saprophytic fungi is affected by biological factors, abiotic factors, and substrate quality, it consists of three basic models: **the relationship model of biological factors**, **the relationship model of abiotic environmental factors**, **the relationship model of matrix mass**.

In terms of **biological factor relationship model,** we established a linear model between mycelium elongation and decomposition rate. Since the decomposition rate is also affected by the strain resistance, we introduce the concept of **importance value** to reflect this effect in the proportional coefficient of the linear model. The important value is the sum of **relative abundance** and **relative dominance**. In this model, we also established the comprehensive enzyme activity-decomposition rate relationship model to describe the relationship between the comprehensive enzyme activity and decomposition rate. Besides, we used the larger **niche width value** to describe the fungus's inadaptability to the environment. In terms of abiotic environmental factors, the local climate will change periodically with time and we mainly take temperature and humidity factors into account. The variation of local climate model can be predicted by the variation of atmospheric trend. In terms of matrix mass factor, the effect of the cumulative loss rate of litter decomposition on the decomposition rate of litter at the present stage was mainly considered. Then, through linear programming model analysis, an optimal solution of decomposition rate is obtained, and a weight of three factors affecting the decomposition rate is obtained. The linear programming model is used to analyze the above three kinds of factors, and a relatively comprehensive relationship model of influence factors is obtained.

We also conducted a sensitivity analysis to verify the stability of the model and the advantages and disadvantages of the model as well as improvement methods are also analyzed. Besides, an article is also attached.

Key Words: the relationship model of influence factors, linear programming model; Important value, Niche width value; Matlab, lingo

Contents

1 Introduction

1.1 Problem Background

The carbon cycle, a material chain within the inorganic environment and the organisms, is of great significance, keeping the balance of the carbon dioxide in the atmosphere. The decomposer, including the bacterium, the fungus, and some kinds of animals, gives the carbon existed in organic matter to the inorganic environment, which is a key approach in the carbon cycle. A vital component of this is the plant material and woody fibers are decomposed by microorganism， especially by fungi[1].

And a paper published recently shows that the growth of the fungus has a close relationship with temperature, humidity and other environmental factors[2].

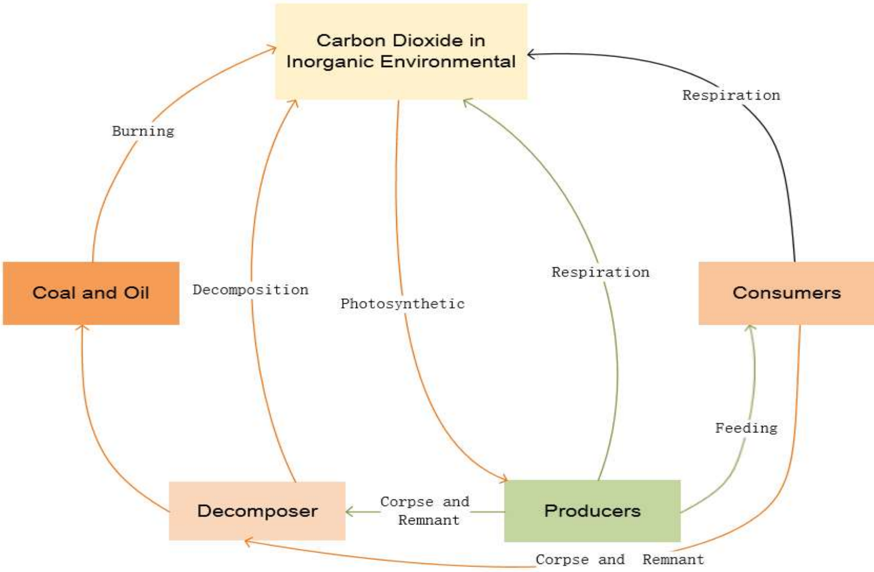

Figure 1: The carbon cycle

1.2 Restatement of the Problem

In this paper, we will make models to tackle with these following five questions.

- Build a mathematical model to depict the breakdown of ground litter and woody fibers in a state where multiple fungi coexist.
- Describe the interaction between the fungi with different growth rate and different moisture tolerances.

- Analyze the interaction in both short-term and long-term trends and examine the sensitivity to rapid fluctuations in the environment. Besides, evaluate the overall impact of changing atmospheric trends to the local climate change.
- Predict the relative advantages and disadvantages for each species, forecast which combination of species will persist and calculate the influence of different environment to decomposition.
- Evaluate the impacts of the diversity of fungal communities and the biodiversity of the local environment.

1.3 Our Approach

Our ideas of modeling are displayed as the flow chart below.

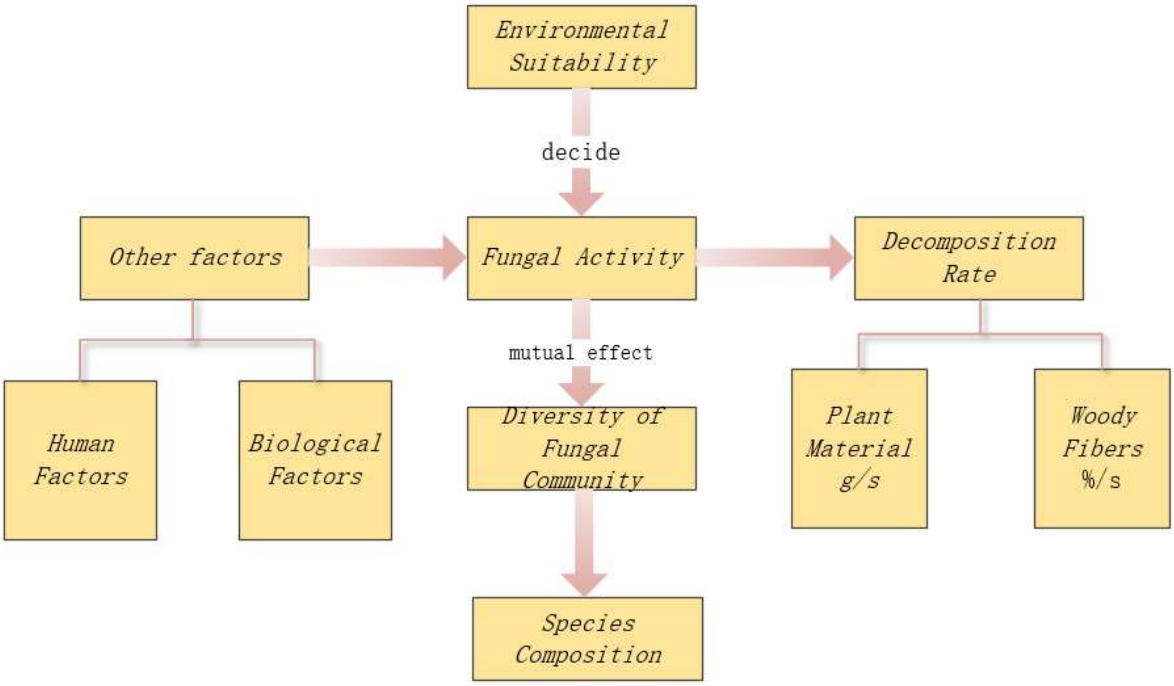

Figure 2: the flow chart 1

We build **the relationship model of influence factors** to solve the whole question, which concludes three basic models: **the relationship model of biological factors**, **the relationship model of abiotic environmental factors**, **the relationship model of matrix mass**. These models give a good answer to the questions.

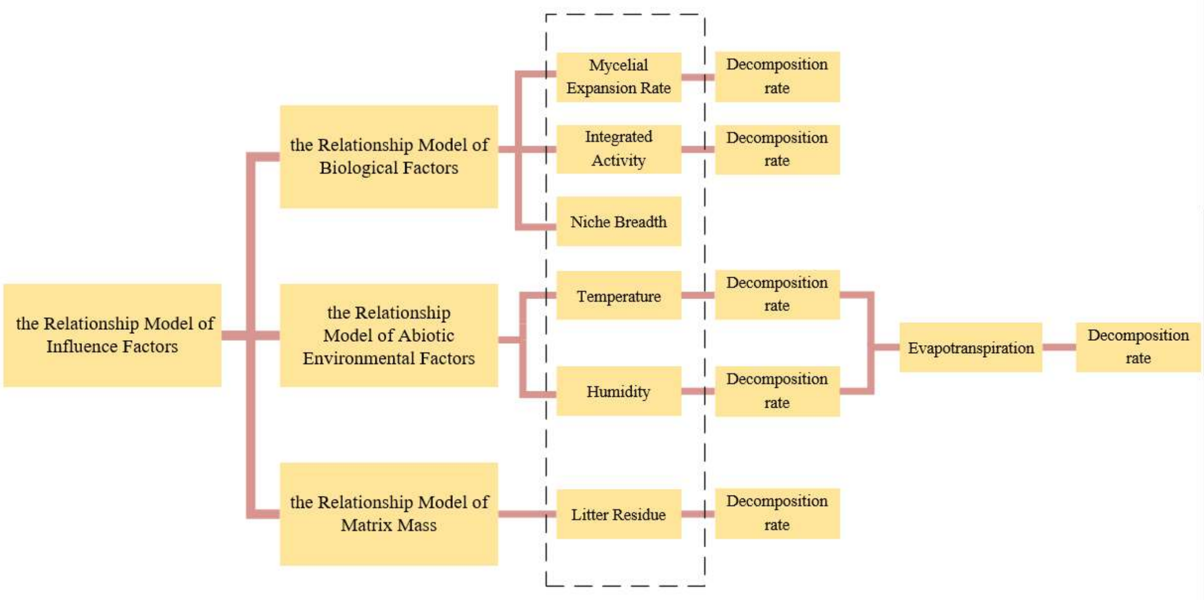

Figure 3: the flow chart 1

2 General Assumptions and Model Overview

2.1 Assumptions

To simplify the given problem, we make the following assumptions for our models.

- The data we acquired are real and reliable. The papers we read are in good quality and more enough to help us get correct conclusion.
- We mainly use the data get in the laboratories, for it is almost impossible to control the temperature or the humidity in the nature. Besides, the breakdown of ground litter and woody fibers are also relative to some bacteria and actinomycetes.
- The single variable principle is strictly followed, the irrelevant interference is eliminated or screened and the irrelevant variables are the same and appropriate, so the relationship we got between the independent variable and the dependent variable is reliable.
- We suppose that the given punch of land is in an area of north America.
- We assume that the ecosystem in this region is relatively stable.
- We focus on the middle stage of wood decomposition, which could represent the whole decomposition process well.
- We regard the activity of enzymes related to the rate of decomposition as constant.
- The mycelium extends equally in all directions and has the same number of spores per unit area.

2.2 Model overview

The decomposition of lignin fibers by saprophytic fungi is affected by biological factors, abiotic factors, and substrate quality. We adopt a **process model**, with time **t** as the most basic independent variable, to describe the process of fungi breaking the down litter, mainly lignocellulose. We describe the model as **the relationship model of influence factors**, which concludes three basic models: **the relationship model of biological factors, the relationship model of abiotic environmental factors, the relationship model of matrix mass**[3].

Description of relationship model of biological factors: In the model of the existence of a variety of fungi, there are differences in enzyme expression types, content, anti-adversity, growth advantages and other aspects of different strains, due to the difference of gene expression at the molecular level, thus showing different decomposition rates. The strains with high mycelial elongation and high competition have high inherent rate of wood decomposition. The effect of enzyme activity on decomposition rate was expressed by a comprehensive enzyme activity decomposition rate model.

Description of relationship model of abiotic environmental factors: The abiotic environmental factors that affect the decomposition of ground litter mainly composed of lignocellulose include climate factors and soil factors, among which the climate factors are mainly manifested in temperature, humidity, rainfall and evapotranspiration, and the relationship model of temperature and humidity is predicted by the overall change of local atmospheric trend. Soil factors are mainly reflected in soil nutrient conditions. This model mainly analyzes the influence of climate factors (temperature and humidity) on the model. Since evapotranspiration is a comprehensive reflection of climate temperature and humidity, we use the relationship between evapotranspiration and decomposition rate to describe the abiotic environmental factor model. Generally, the decomposition rate of litter is the highest under warm and humid conditions.

Description of relationship model of matrix mass: Matrix mass refers to the physical and chemical properties of ground litter itself, which is the internal factor that restricts the decomposition of ground litter. As the quality of matrix will change with the decomposition process, the relationship model can be divided into stage model and continuous model. We apply the continuous model, so we mainly consider the effect of residual lignin quality on decomposition rate.

3 Model Preparation

3.1 Notations

Critical notations employed in this paper are displayed in Table 1.

Table1: Notations

Symbol	Description	Unit
t	Time	h or d
T	Temperature	℃
W	Humidity	%
N	Species of fungi	\
MR	Mycelial expansion rate (Hyphal extension rate)	mm/d
G	Growth rate of fungi	\
PA	Acid phosphatase activity	U, active unit
EEA	Extracellular enzymes activity	U, active unit
IA	Integrated activity	U, active unit
EA	enzymatic activity	U, active unit
CR	Competitive ranking	\
WEW	Water ecological width	\
TEW	Temperature ecological width	\
Ba	Niche breadth	\
I	Important value	\
Ra	Relative abundant	%
Ev	Evapotranspiration	mm
LR	Litter residue	%
CA	Relative dominance	\
DML	Decomposition loss law	%
CML	Cumulative dry weight loss rate	%
DR	Decomposition rate	%/a or %/d or %/h
AD	Annual decomposition	Kg/a
AL	Annual litter	Kg/a

3.2 New concept definition

- **Mycelial expansion rate[4] (Hyphal extension rate)**: To characterize the growth of each isolate, we quantified its hyphal extension rate and hyphal density. In brief, we inoculated the center of five replicate Petri dishes (plates) per isolate with a 5-mm-diameter plug of sample culture. Plates were incubated for 2 wk or until the growing isolate reached the edge of the plate. Hyphal extension rate was quantified as the linear extension rate in millimeters per day. Hyphal density was measured using a similar design with media covered by a layer of cellophane, which allowed us to remove and weigh the mycelium. We quantified hyphal density as micrograms dry mass per cubic centimer at 1 cm from the edge of the growing front.
- **Competitive ranking[4]**: To quantify the ability of our fungal isolates to displace other isolates in direct combat, we conducted pairwise competition trials with all 20 species, excluding intraspecific interactions. We inoculated plates with two straight lines composed of three plugs per isolate, facing each other at 2 cm from the center of the plate. Inoculation time was

adjusted to known differences in extension rate, such that both species formed a hyphal front across half the plate 1 cm apart at the start of the trial. Plates were incubated for up to 8 wk, until one fungus completely displaced the other (competitive exclusion), or when no displacement was observed for 3 wk (deadlock). Based on the outcome of 615 unique trials, we quantified three traits at the species level. “Competitive ranking” is the position of a species in the overall competitive hierarchy among our fungi, calculated using the Elo ranking system. “Offensive ability” is the average extension rate of a fungus when displacing its competitor, divided by its extension rate in monoculture. Finally, “defensive ability” is the average rate at which a fungus is overgrown, divided by the competitor’s extension rate in monoculture.

- **Humidity**[5]: Surface air humidity refers to the air humidity at the height specified by surface meteorological observation (i.e. 1.25-2.00 m, 1.5 m in China). It is measured by the dry and wet bulb thermometer and hygrometer installed in the louver box (the basic station regularly observes 5 times a day, and the reference station regularly observes 24 times a day).
- **Integrated activity**[3]: Among all enzymes, acid phosphatase had the strongest negative correlation with decomposition rate, while cellulase had the strongest positive correlation with decomposition rate. Thus, we proposed the concept of integrated enzyme activity to show the effect of enzyme activity on the decomposition rate. Within this definition, the activity of positively related enzyme was positive, and the activity of negatively related enzyme was negative, represented by EA.
- **Decomposition loss law**[4]: The proportion of litter mass decomposed by fungi in total litter mass.
- **Relative dominance**[6]: Relative abundance is an index to estimate the proportion of individual number of species in the total individual number of species, which is mostly used in the field investigation of community. The relative abundance is expressed by the area ratio of each strain in the colony.
- **Litter residue**[7]: Litter residue before sampling time.
- **Evapotranspiration**[7]: As an important part of energy balance and water cycle, evapotranspiration not only affects the growth and yield of plants, but also affects the atmospheric circulation and plays a role in regulating climate.
- **Niche breadth**[6]: The criteria for measuring the adaptability of species to the environment are calculated based on the important values of various groups.
- **Important value**[6]: Important value is the sum of Relative abundant and Relative dominance.
- **Temperature and moisture niche**[4]: Physiological response curves have previously been described for our fungi in ref. 25. Skew-normal distribution models were fit to the extension rate of each isolate across a temperature (10 ℃ to 40 ℃, five replicates per isolate) and moisture (water potentials -0.5 to -4.5 MPa, three replicates) gradient, with extension rate measured at six values along each gradient. To quantify temperature and moisture niche traits of each isolate based on these response curves, we derived the niche width, minimum, and maximum (marking the range of conditions that supports at least half the maximum extension rate of the fungus), as well as the optimal temperature and moisture conditions (where maximum extension rate is reached).

3.3 Data Collection

Due to the fact that we have no access to experiment on this problem by ourselves, the data we collected and used in this paper are not the first-hand data. They are from the papers published before and we will give clear indication of the source of the data in the following table.

Table 2: The source of the data

Article of paper	Author of paper
A trait-based understanding of wood decomposition by fungi	Lustenhouwer, N. et al.
Diversity of endophytic fungi in Chinese fir leaves and their effects on litter decomposition	Ming-rong C.

4 Analysis and models

4.1 General Statement

Litter decomposition is a complex degradation process, affected by biological factors, abiotic factors, matrix quality and other factors. Among them, biological factors include fungal species and fungal community composition, abiotic factors mainly include soil and climate. Meanwhile, the so-called decomposition rate of litters is always only a reflection of the decomposition status of litters under a specific space-time condition. Ultimately, different factors affect the rate of decomposition by acting on the fungus itself.

Through extensive literature review, we know that the rate of fungal hyphae expansion is the strongest single predictor of the rate of fungal decomposition of wood fiber. Besides, the growth rate of fungi is reflected by the expansion rate of mycelia. Thus, we can establish contact between the growth rate and the lignin fiber expansion rate.

We adopt the linear model as the description of the relationship between Mycelial expansion rate (MR) and Decomposition rate (DR)[4].

$$DR = \sum_{n=1}^{n\to\infty} I_1 MR_1 + I_2 MR_2 + I_3 MR_3 + \cdots + I_n MR_n$$

Important value = Relative abundant + Relative dominance[6]

$$I = RA + CA$$

We give value to the competitive ranking from -1 to 1. We obtain a linear relationship by exploring the relationship from the competitive abilities of different strains to their decomposition rate. We use slope to represent the value of relative dominance CA,

$$CA \in (-1,1)$$

When the number of strains was 1, the relative dominance was 0.

Relative abundance is an index to estimate the proportion of individual number of species in the total individual number of species, which is mostly used in the field investigation of community. The relative abundance is expressed by the area ratio of each strain in the colony:

$$RA(\%) = \left(\frac{A\ Colony\ diameter}{Total\ colony\ diameter}\right)^2 \times 100\%,\ \ RA \in [0,1]$$

In the same colony structure at the same time, the relative abundance and relative dominance of each kind of fungi were determined. The short-term dominant species composition can be obtained through the optimal solution of linear programming.

4.2 Relationship Model of Biological Factors

As for biological factors, apart from mycelial expansion rate, there were also competition among strains that affected the decomposition rate. The competition of species can be reflected in relative abundance, relative dominance and enzyme expression. Among all enzymes, acid phosphatase had the strongest negative correlation with decomposition rate, while cellulase had the strongest positive correlation with decomposition rate.

Thus, we proposed the concept of integrated enzyme activity to show the effect of enzyme activity on the decomposition rate. Within this definition, the activity of positively related enzyme was positive, and the activity of negatively related enzyme was negative, represented by EA.

The experience model of ***Decomposition loss law - enzymatic activity*:**

Decomposition loss law, DML (%) = ke^{IA}, k is Undetermined coefficient[4].

$$IA=\sum_{n=1}^{\infty} EA_n = EA_1 + EA_2 + \cdots + EA_n,\ EA \in \begin{cases} > 0,\ \ Positive\ correlation\ enzyme \\ < 0, Negative\ correlation\ enzyme \end{cases}$$

For the prediction of the relative advantages and disadvantages of each species and the possible persistent species combination, the **Hurlbert niche width** value shows that the larger the **Hurlbert niche width** value is, the weaker its ability to adapt to the environment is.

Hurlbert niche width[6]:

$$Ba = \frac{Ba' - 1}{r - 1}$$

$$Ba' = \frac{1}{\sum_{j=1}^{r}\left(P_{ij}^2\right)}$$

$$P_{ij} = \frac{n_{ij}}{N_i}, N_i = \sum_{j=1}^{r} n_{ij}$$

Besides, P_{ij} is the utilization ratio of strain i to resource j, n_{ij} is the important value of strain i, and r is the number of resource grades with Ba' limited to [0, logr].

4.3 Relationship Model of Abiotic Environmental Factors

In terms of abiotic environmental factors, the local climate will change periodically with time and we mainly take temperature and humidity factors into account. The variation of local climate model can be predicted by the variation of atmospheric trend.

And different temperature and humidity will affect the decomposition rate of the same fungal community. Under the condition of high humidity, the strains with strong moisture tolerance played a leading role, otherwise, the strains with strong drought tolerance played a leading role. At higher temperatures, the strains resistant to high temperature play a dominant role, and vice versa, the strains resistant to low temperature play a dominant role.

Therefore, in terms of abiotic factors, it is necessary to explore the influence of factors that do not periodically change with time, such as temperature and humidity, on the total decomposition rate of bacterial colonies.

4.4 Relationship Model of Matrix Mass

In terms of matrix mass factor, the effect of the cumulative loss rate of litter decomposition on the decomposition rate of litter at the present stage was mainly considered. The cumulative dry weight loss rate and decomposition rate of litter decomposition process are calculated as follows[7]:

Cumulative dry weight loss rate, $CML(\%) = \frac{(M_0 - M_t)}{M_0} \times 100\%$

In the formula, M_0 represents the initial dry weight of litters and M_t represents the residual dry weight of litters after t days' decomposition.

$$e^{-kt} = \frac{X_t}{X_0}$$

In the formula, X_0 represents the initial dry weight of litters and X_t represents the residual dry weight of litters after t years' decomposition.

Then, through linear programming model analysis, an optimal solution of decomposition rate is obtained, and a weight of three factors affecting the decomposition rate is obtained.

$$DR(\xi, \nu, \zeta) = \alpha\xi + \beta\nu + \gamma\nu, \xi = \xi(t), \nu = \nu(t), \zeta = \zeta(t).$$

Within the formula, DR is decomposition rate, ξ is a function of biological factor over time t, ν is a function of abiotic factor over time t, and ζ is a function of matrix mass over time t.

The linear programming model is used to analyze the above three kinds of factors, and a relatively comprehensive relationship model of influence factors is obtained.

5 Results

5.1 Task 1

We established a biological factor model to describe the decomposition of litter and lignocellulose in the presence of various fungi. In the biological factor model, the types and contents of enzymes expressed by different strains are different. The enzymes we consider mainly include cellulose decomposing enzymes, phosphatase and extracellular enzymes.

The expression of cellulose decomposing enzymes directly affects the decomposition rate of fungi, and phosphatase and extracellular enzymes indirectly affect the decomposition rate of fungi by affecting the growth rate of fungi. The mycelial elongation is used to represent the growth rate of fungi, and the decomposition rate of fungi has a linear relationship with the mycelial elongation:

$$DR = \sum_{n=1}^{n\to\infty} I_1 MR_1 + I_2MR_2 + I_3MR_3 + \cdots + I_nMR_n$$

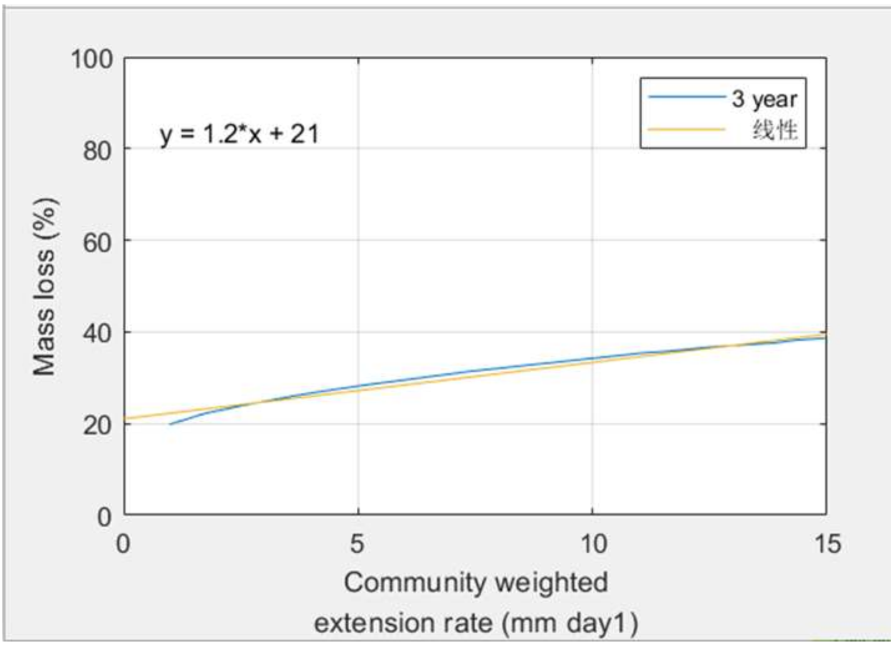

Figure 3

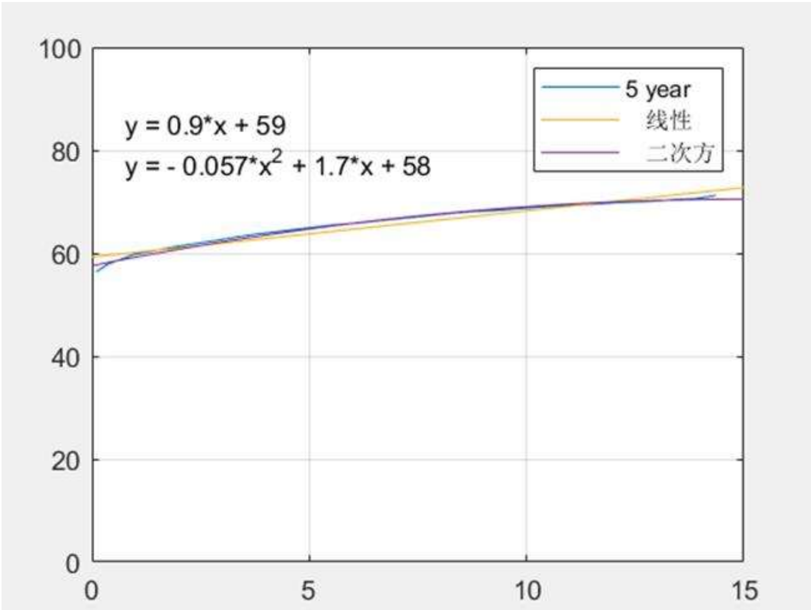

Figure 4

The scale coefficient reflects the interaction of different kinds of fungi. The fungi with strong stress resistance and strong competition showed the advantage of decomposition ability and the disadvantage of growth ability, while the fungi with weak stress resistance and weak competition showed the advantage of growth ability and the disadvantage of decomposition ability.

5.2 Task 2

In the analysis of the short-term trend of the interaction and species composition of different fungal colonies, we do not consider the changes of environmental conditions such as temperature and humidity with time. We mainly analyze the interaction and species composition of different fungal colonies through the relationship model of biological factors. In the biological factor relationship model, we focus on the change of the important value. Through the optimal solution of linear programming, we can get the maximum of the overall decomposition ability of the colony in theory, and can get the corresponding proportion of the relative abundance of each fungal colony and the distribution of the relative dominance, so as to draw the conclusion of the interaction of various fungi in the colony in a short time. If the overall decomposition ability of the colony is far less than the optimal solution, then the colony has a large space for development.

For the long-term trend analysis, we mainly consider the impact of abiotic environmental factors on the interaction between different fungal colonies and the composition of colony species. In a long time, abiotic environmental factors such as temperature and humidity will have relatively large fluctuations. However, due to the different environmental adaptability of different strains, the species composition of colonies may change accordingly. At this time, Hurlbert niche width value was used to infer the change trend of the colony.

Hurlbert niche width can be defined as follows,

$$Ba = \frac{Ba' - 1}{r - 1}$$
$$Ba' = \frac{1}{\sum_{j=1}^{r}\left(P_{ij}^{2}\right)}$$
$$P_{ij} = \frac{n_{ij}}{N_i}, N_i = \sum_{j=1}^{r} n_{ij}$$

P_{ij} is the utilization ratio of strain i to resource j, n_{ij} is the important value of strain i, and r is the number of resource grades with Ba' limited to [0, logr].

The larger the Hurlbert niche width value is, the weaker the ability of the fungus colony to adapt to the environment is. In order to reduce the interference of many uncertain factors, we describe the periodic changes of local temperature and humidity to determine the change regular pattern of local weather model. By analyzing the changes of colony decomposition ability under different temperature and humidity, we evaluated the sensitivity of rapid fluctuation of local environment. If the decomposition capacity does not change greatly, it indicates that the sensitivity of rapid fluctuation of local environment is weak; if the decomposition capacity changes greatly, it indicates that the sensitivity of rapid fluctuation of local environment is strong.

5.3 Task 3

In order to reduce the interference of many uncertain factors, we describe the periodic changes of local temperature and humidity to determine the change law of local weather model. By analyzing the changes of colony decomposition ability under different temperature and humidity, we evaluated the sensitivity of rapid fluctuation of local environment. If the decomposition capacity does not change greatly, it indicates that the sensitivity of rapid fluctuation of local environment is weak; if the decomposition capacity changes greatly, it indicates that the sensitivity of rapid fluctuation of local environment is strong.

5.4 Task 4

By comparing the decomposition rates of different fungal communities, it can be concluded that the more diverse fungal communities are, the more stable the local environment is. The importance of biodiversity to local carbon cycle can be concluded by comparing the decomposition rates of fungal communities with different diversity at different times.

5.5 Task 5

By comparing the corresponding decomposition rate differences under different fungal community diversity, it can be concluded that the more diverse the fungal community is, the more important it is to maintain the local environment stability.

Meanwhile, by comparing the decomposition rates of different fungal communities in different time periods, we can conclude the importance of biodiversity to local carbon cycle.

5.6 Ultimate Results

The enzyme activity changes dynamically during the decomposition of apoplast due to the competition between microorganisms. We used the elongation of mycelium to express the change of enzyme activity. In the process of solving, we recorded that the elongation rate of each mycelium multiplied by their corresponding relative multiplicity and then compared to the sum of the critical values is less than half of the slope of the line of elongation rate and decomposition rate.
We can thus obtain the following constraints for this colony.

That is

We substitute the change of constraints into the original linear equation and use Lingo to solve it.

The Lingo code is as follows:

```
max=0.6425*x1+0.627*x2+0.043*x3+0.0865*x4+0.10*x5+0.043*x6+0.0865*x7+0.1295*x8+0
.243*x9;
x1+x2+x3+x4+x5+x6+x7+x8+x9<=15;
x1+x2+x3+x4+x5+x6+x7+x8+x9>10.6;
x1>1;x2>1;x3>1;
x4>1;x5>1;x6>1;
x7>1;x8>1;x9>1;
end
```

The calculation results show as follows.
"OBJECTIVE VALUE) 5.856000" means that the optimal objective value is 5.856000.
"VALUE" gives the value of each variable in the optimal solution: x1=7.000000, x2=1.000000, x3=1.000000, x4=1.000000, x5=1.000000, x6=1.000000, x7=1.000000, x8= 1.000000, x9=1.000000.

The meaning of "REDUCED COST" is: the REDUCED COST value of the base variable is 0. For non-base variables, the corresponding REDUCED COST value indicates the amount by which the target function decreases when the non-base variable increases by one unit (other non-base variables remain unchanged). All nine variables in this example are base variables.
"SLACK OR SURPLUS" gives the values of the slack (or residual) variables, indicating whether the constraint takes the equation constraint; the slack variables in lines 2 and 5 to 11 are all 0, indicating that for the optimal solution, both constraints take the equation constraint; the slack variables in lines 1, 3, and 4 are not 0, indicating that for the optimal solution The relaxation variables in lines 1, 3 and 4 are not 0, indicating that for the optimal solution, this constraint takes an inequality constraint.

The "DUAL PRICES" constraint is the value of the shadow price (also known as the pairwise price): rows 2, 3, and 4 (constraints) correspond to shadow prices of 5.000000, 4.000000, and 6.000000, respectively.

```
Global optimal solution found.
Objective value:                              5.856000
Infeasibilities:                              0.000000
Total solver iterations:                             0
Elapsed runtime seconds:                          0.05

Model Class:                                        LP

Total variables:                    9
Nonlinear variables:                0
Integer variables:                  0

Total constraints:                 12
Nonlinear constraints:              0

Total nonzeros:                    36
Nonlinear nonzeros:                 0


                                Variable           Value        Reduced Cost
                                      X1        7.000000            0.000000
                                      X2        1.000000            0.000000
                                      X3        1.000000            0.000000
                                      X4        1.000000            0.000000
                                      X5        1.000000            0.000000
                                      X6        1.000000            0.000000
                                      X7        1.000000            0.000000
                                      X8        1.000000            0.000000
                                      X9        1.000000            0.000000

                                     Row    Slack or Surplus      Dual Price
                                       1        5.856000            1.000000
                                       2        0.000000            0.6425000
                                       3        4.400000            0.000000
                                       4        6.000000            0.000000
                                       5        0.000000           -0.1550000E-01
                                       6        0.000000           -0.5995000
                                       7        0.000000           -0.5560000
                                       8        0.000000           -0.5425000
                                       9        0.000000           -0.5995000
                                      10        0.000000           -0.5560000
                                      11        0.000000           -0.5130000
```

Figure 5

6 Model Evaluation and Further Improvements

6.1 Sensitivity Analysis

The aim of our sensitivity analysis is to find out over which range the parameters change with no change in the optimal substrate. The linear programming models we use assume that the parameters are known or deterministic, however it is sometimes difficult to know exactly these parameters, which also change accordingly with changes in biotic factors, abiotic factors (soil and climate), substrate quality, etc., and eventually lead to changes in the optimal solution of the model.

The following figure shows the sensitivity analysis window of Lingo.

: range

Ranges in which the basis is unchanged:

Objective Coefficient Ranges:

Variable	Current Coefficient	Allowable Increase	Allowable Decrease
X1	0.6425000	INFINITY	0.1550000E-01
X2	0.6270000	0.1550000E-01	INFINITY
X3	0.4300000E-01	0.5995000	INFINITY
X4	0.8650000E-01	0.5560000	INFINITY
X5	0.1000000	0.5425000	INFINITY
X6	0.4300000E-01	0.5995000	INFINITY
X7	0.8650000E-01	0.5560000	INFINITY
X8	0.1295000	0.5130000	INFINITY
X9	0.2430000	0.3995000	INFINITY

Righthand Side Ranges:

Row	Current RHS	Allowable Increase	Allowable Decrease
2	15.00000	INFINITY	4.400000
3	10.60000	4.400000	INFINITY
4	1.000000	6.000000	INFINITY
5	1.000000	6.000000	1.000000
6	1.000000	6.000000	1.000000
7	1.000000	6.000000	1.000000
8	1.000000	6.000000	1.000000
9	1.000000	6.000000	1.000000
10	1.000000	6.000000	1.000000
11	1.000000	6.000000	1.000000
12	1.000000	6.000000	1.000000

: |

Figure 6

The original significant value coefficient of the x1 variable in the objective function is 0.6425, Allowable Increase = $+\infty$, Allowable Decrease = , indicating that the optimal base remains constant when it varies in the range [60- ,   ]. For the other variables, a similar interpretation is possible. Since the constraint does not change at this time (only the coefficient of some important value in the objective function changes), the optimal base remains unchanged means that the optimal solution remains unchanged (of course, the optimal value changes because the coefficient of important value in the objective function changes).

The right hand side of the constraint in line 2 (abbreviated as RHS) is originally 15, and the optimal base remains unchanged when it changes in the range [15.00000, $+\infty$]. Lines 3, 4, and 5 can be interpreted similarly. However, since the constraint changes at this time, the optimal base, even if it remains unchanged, the optimal solution of the colony importance value and the optimal value of the mycelial extension rate will change.

The results of the sensitivity analysis indicate the range of coefficients for which the optimal base remains constant. From this, it is also possible to further determine how the optimal base of the decomposition rate and the optimal solution of the colony importance value and the optimal value of the mycelial elongation rate change when the decomposition rate coefficient of the objective function and the right end term of the constraint change in a small way.

6.2 Strengths

- The model is very comprehensive and specific, taking into account biological factors, abiotic factors, matrix quality and other factors. Besides, It's more reflective of the real situation.
- Only time **t** is used as the basic variable, so it is quite convenient to be applied in other situation.
- It reveals the decomposition mechanism of litters from the micro and macro levels, which is scientific.
- Linear models give more accurate results.

6.3 Weaknesses and Further Improvements

6.3.1 Weaknesses

- For the reason that our data is not first-hand, so there is the possibility of falsehood. The data is not sufficient enough since we do not have the access to all data we need, so we have not tested the model effectively.
- The relationship between abiotic factors and biological factors was not established effectively, and the correlation degree of the relationship model of the three factors could not be analyzed.

6.3.2 Further Improvements

- If time is sufficient and conditions permit, we can carry out experiments to collect data ourselves, ensuring the authenticity. If we have access to huge amounts of data, the model we built will be more accurate.
- If the data is efficient, we could deal with the relationship with the different factors well.

7 An article on the effects of fungi in ecological of systems

Ecosystem Overview

Ecosystem refers to the unity formed by organisms and the environment in a certain space of nature. In this unity, organisms and the environment influence and restrict each other and maintain a relatively stable dynamic balance in a certain period of time. An ecosystem is composed by non-living matter and energy, producers, consumers, and decomposers, having the function of material circulation, energy flow and information transmission. As an important link in the material cycle, the material must be returned from the biological world to the inorganic environment through the decomposition of the decomposer.

Overview of Fungal Action

Since most of the earth is covered with vegetation, litter accounts for a large proportion of the organic matter to be decomposed. Meanwhile, the main components of litter are cellulose, hemicellulose and lignin, whose decomposition mainly depend on some saprophytic fungi. Therefore, the decomposition of lignocellulose by saprophytic fungi is of great importance to material recycling. In this article, we will focus on the role of fungi in ecosystems.

Fungi species are diverse and have a wide range of impacts on the ecosystem. For example, mycorrhizal fungi affect the competitive balance among plants, thus affecting the community biodiversity[8]. It is worth noting that fungi play a crucial role in the degradation of leaf litter.

Saprophytic fungi have succession in the process of litter decomposition. Because of the rapid spore germination and hyphae growth, sugar fungi become the dominant bacteria at the early stage of material decomposition. However, with the accumulation of metabolic by-products, the growth conditions become unsuitable and they enter the dormant period. The next stage of succession is cellulose degradation by fungi. After their extinction, lignin degradation is carried out by Basidiomycetes higher[9].

Thus, fungi are crucial for lignocellulosic decomposition. But more than one type of fungus is known to degrade lignocellulose, so what is the relationship between these fungi? What factors affect the degradation process simultaneously?

The Relationship Between these Saprophytic Fungi

The decomposition of litter and wood fibers on the ground involves a variety of fungi with different growth rates and different moisture tolerance. Among them, the expression of cellulolytic enzymes directly affected the decomposition rate of fungi, while phosphatase and extracellular enzymes indirectly affected the decomposition rate of fungi by affecting the growth rate of fungi. The results showed that the fungi with strong stress resistance and competition showed the advantage of decomposition ability and the weakness of growth ability. However, fungi with weak resistance to adversity and competition reflect the advantages of growth ability and weak decomposing ability.

Through middle school, we know that fungi do not compete with each other when resources such as food and space are plentiful. Indeed, previous studies have shown that there are complex relationships between different types of fungi. In some cases, it is synergistic and in others it is competitive. For example, in this newly published study[10], we see that there is no antagonism between the two different experimental strains, but synergism between the two strains increases the activity of lignin degradation enzymes, thus increasing the lignin degradation rate. The study[11], published in the He Journal of General and Applied Microbiology, also shows that growing a mixture of certain fungi breaks down cellulose more efficiently than growing it alone. But there is no doubt that competition between and within species due to limited resources is common. In the short term, it is possible that species do not compete with each other because of abundant resources, and may even cooperate. But in the long run, they must have the possibility to compete for resources. However, it is also possible that different species of fungi have symbiotic relationships, in which they need to cooperate in order to survive longer together and produce offspring when conditions are harsh.

Interaction between fungi and their environment

In a relatively stable ecosystem, the number of species and the relative dominance of species generally do not undergo drastic changes. However, when the atmospheric environment changes, biological factors will also change with it. Since the rate of atmospheric change is relatively slow, we pay attention to the overall trend of atmospheric change. With the fluctuation of environmental factors and conditions, the species and number of fungi and other indicators will show a certain fluctuation, the fluctuation has a certain lag. However, combined with a large number of statistical data analysis, these biological indicators can directly reflect the fluctuation of environmental conditions, at least can reflect whether the environmental conditions are suitable for the growth and reproduction of organisms in a period of time. Now in the stage of global warming, the climate is getting drier, so the growth of the hygrophilic fungi will be inhibited to a certain extent.

According to the theory of natural selection, we know that possible species and combinations of species are likely to persist and must be better adapted to their environment. So the so-called comparative advantages and disadvantages, that is, the ability to adapt to the local climate conditions, climate conditions are likely to change, may even be changeable. In this sense, populations that have more mutations or are more prone to mutations are more likely to gain and maintain competitive advantages. However, when the climate conditions are relatively stable in the short term, the population that can stably inherit and accumulate beneficial variation will undoubtedly have a competitive advantage. In arid and semi-arid regions, the fungi with good water tolerance are in the competitive advantage, so they can survive better while in tropical rain forests, hygrophilic fungi have a greater survival advantage, access to more resources, and are therefore more likely to exist.

The greater the number and species of fungi, the stronger the community diversity, the higher the overall decomposition efficiency for litter, the greater the contribution to the carbon cycle, and the more conducive to maintaining the stability of the local ecosystem. Different fungi may decompose different components of litter, and the order of decomposition may be different.Fungi can also interact with plant roots, affect plant growth, and participate in the formation and maintenance of food chains and food webs. Therefore, communities with high biodiversity not only have higher decomposition efficiency, but also have more stable ecosystems due to more complex nutrient structures.

Fungi play a very important role in the ecosystem, especially as decomposers to participate in the decomposition of plant material and woody fibers, which is of great significance to carbon cycle. It ensures the smooth circulation between the inorganic environment and the organic biological community. In the process of degradation, a variety of fungi cooperate or antagonize each other to decompose dead branches and leaves. The degradation process is also affected by environmental factors and substrate quality.

8 References

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9. Appendix Codes

Matlab：

```
c=-[0.6425,0.627,0.043,0.0865,0.10,0.043,0.0865,0.1295,0.243]';

A=[1,1,1,1,1,1,1,1,1;-1,-1,-1,-1,-1,-1,-1,-1,-1];
b=[1,15]';
Aeq=[1,1,1,1,1,1,1,1,1];
beq=[30];
VLB=[1,1,1,1,1,1,1,1,1]';
VUB=[15,15,15,15,15,15,15,15,15]';
L=[0, 0];
[x,fmin,exitflag,output]=linprog(c,A,b,Aeq,beq,VLB,VUB);
Pmax=-fmin
fprintf("x=%.3f\n",x);
```